

Supporting Videos

Video S1. Urination of a rat, mass of 0.24 kg. Time slowed by 33 X.

Video S2. Urination of a goat, mass of 70 kg. Time slowed by 17 X.

Video S3. Urination of a cow, mass of 640 kg. Time slowed by 33 X.

Video S4. Urination of an elephant, mass of 3540 kg. Time slowed by 33 X.

Appendix

Unsteady hydrodynamic urination model for large and small animals

In this section, we estimate the time for the urine flow to achieve steady state. The column of urine descends due to gravitational and bladder pressure forces. This descent is resisted by viscosity, fluid inertia and capillary pressure.

At time $t = 0$ the urethra is empty. We parametrize the height of urine in the urethra with a height $z(t)$, measured from the bladder. Consider **Figure 1E** in the main text. We consider a control volume including the mass of urine with the bladder and urethra. Conservation of momentum for this control volume may be written

$$(m + m_a) \ddot{z} = A (P_{\text{bladder}} + P_{\text{gravity}} - P_{\text{inertia}} - P_{\text{viscosity}} - P_{\text{capillary}}). \quad (1)$$

Each terms in Equation (1) has units of force and may be written simply. The mass of urine in the urethra is $m = \rho\pi\alpha D^2 z/4$. Using techniques presented by Bush*, the added mass associated with acceleration of fluid in the bladder may be written $m_a = 7\rho\pi\alpha^{3/2}D^3/48$. Bladder pressure P_{bladder} is constant. Hydrostatic pressure P_{gravity} scales as $\rho g z$ where g is the gravitational acceleration. Dynamic pressure P_{inertia} scales as $\rho \dot{z}^2/2$, and is associated with inertia of the flow. Assuming laminar flow, the pressure drop due to viscosity in a long cylindrical pipe is $P_{\text{viscosity}} = 32\mu z \dot{z}/\alpha D^2$. The capillary force of drops generated from an orifice of effective diameter $\sqrt{\alpha}D$ is $P_{\text{capillary}} = 4\sigma/\sqrt{\alpha}D$. Substituting these terms into Equation (1), we arrive at

$$\ddot{z} \left(z + \frac{7}{12}\sqrt{\alpha}D \right) = \frac{P_{\text{bladder}}}{\rho} + gz - \frac{\dot{z}^2}{2} - \frac{32\mu\dot{z}z}{\rho\alpha D^2} - \frac{4\sigma}{\rho\sqrt{\alpha}D} \quad (2)$$

We compute velocity \dot{z} in Equation (2) using a Runge-Kutta single-step solver (ode45) in Matlab. Inputs to this equation include allometric relationships of bladder pressure, urethral diameter, and shape factor, given in **Table 1**. **Figure S1** shows the time course of urine velocity. For animal lighter than 100 kg, the flow reaches 90% of its final velocity in 4 seconds, which is 15% of the the time to empty the bladder, 21 seconds. We thus conclude our steady state model reported in the main text is accurate for animals lighter than 100 kg. For larger animals such as elephants, the transition can be substantial.

*Bush JWM (2010) *18.357 Interfacial Phenomena, Fall 2010*. Available at <http://ocw.mit.edu> Accessed 2 May, 2014.

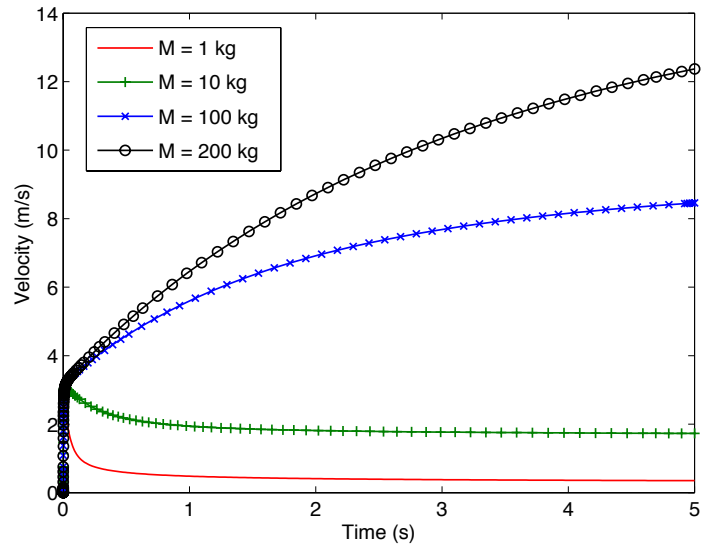


Figure S1. Time course of urine velocity.

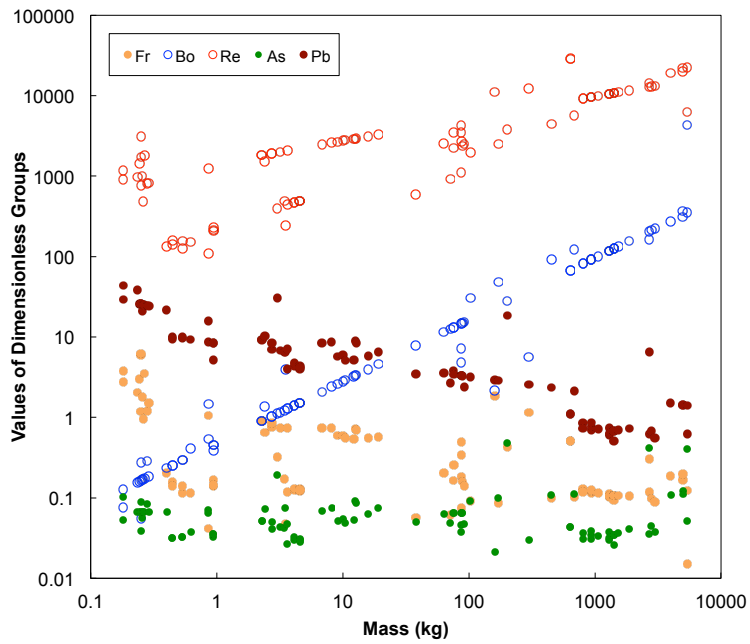


Figure S2. Values of dimensionless groups including the Froude Fr, Bond Bo, Reynolds Re, aspect ratio As, and ratio of bladder and hydrostatic pressure Pb.

Table S1. Duration of urination.

Animal	Sex	Mass (kg)	Duration (s)	Source
Mouse (N=5)	F	0.03	1.50	Experiment at Georgia Tech
Bat	F	0.03	0.32	K. Breuer and S. Swartz, Brown University
Rat (N=5)	F	0.20	0.10	Experiment at Georgia Tech
Applehead Chihuahua	M	3	4	Experiment at local park
Cat	F	5	18	Youtube, Mattern (2000)
Goat	F	18	8	Experiment at Zoo Atlanta
Goat	M	67.8	9	Experiment at Zoo Atlanta
Great Dane	M	71	24	Youtube, Wilcox (1997)
Great Dane	M	71	17	Youtube, Wilcox (1997)
Great Dane	M	71	17	Youtube, Wilcox (1997)
Jaguar	F	76	15	Youtube, Wilson (2001)
Gorilla	F	100	20	Youtube, Miller (1997)
Panda	F	125	11	Youtube, Brown (1996)
Donkey	M	140	8	Youtube, Starkey (1992)
Lion	M	200	36	Youtube, Nowak (1999)
Tapir	F	318	9	Youtube, Lynette (2013)
Tapir	M	318	59	Youtube, Lynette (2013)
Elk	M	331	46	Youtube, Linnaeus (1758)
Zebra	M	430	8	Youtube, Kingdon (1988)
White horse	F	470	10	Youtube, Marvin (1992)
Horse	M	470	17	Youtube, Marvin (1992)
Race Horse	M	600	27	Youtube, Marvin (1992)
Horse	M	850	19	Youtube, Bongiani (1988)
Bison	M	907	20	Youtube, Potts (1997)
Indian Rhinoceros	M	2150	20	Youtube, Toon (2002)
Rhino	F	2200	49	Youtube, Toon (2002)
Rhino	F	2200	12	Youtube, Toon (2002)
Rhino	M	2200	17	Youtube, Toon (2002)
Elephant	F	3538	28	Experiment at Zoo Atlanta
Elephant	F	5000	17	Youtube, Shoshani (1982)
Elephant	F	5000	17	Youtube, Shoshani (1982)
Elephant	F	5000	15	Youtube, Shoshani (1982)
Elephant	M	8000	35	Youtube, Shoshani (1982)
Elephant	M	8000	29	Youtube, Shoshani (1982)
Elephant	M	8000	22	Youtube, Shoshani (1982)

Table S2. Urethral length.

Animal	Sex	Mass (kg)	Length (mm)	Source
Mouse	F	0.02	10	Clair (1999)
Wister Rat (N=20)	F	0.2	9.5	Chang (2000)
Rat	F	0.2	17	Clair (1999)
Sprague-Dawley Rat (N=61)	F	0.3	20.0	Kamo (2004)
Dunkin Hartley Guinea Pig	M	0.4	20.0	Vogel (2007)
Normal Adult Cat	F	2.3	49.4	Johnston (1985)
Normal Adult Cat	F	2.3	50.0	Johnston (1985)
Normal Adult Cat	F	2.7	66.7	Johnston (1985)
Normal Adult Cat	F	2.7	54.2	Johnston (1985)
Normal Adult Cat	F	3.2	68.3	Johnston (1985)
Normal Adult Cat	F	3.6	65.6	Johnston (1985)
Normal Adult Cat	M	3.6	116.6	Johnston (1985)
Normal Adult Cat	M	4.1	107.8	Johnston (1985)
Normal Adult Cat	M	4.1	99.4	Johnston (1985)
Normal Adult Cat	M	4.6	108.3	Johnston (1985)
Normal Adult Cat	M	4.6	118.6	Johnston (1985)
Normal Adult Cat	M	4.6	113.8	Johnston (1985)
Normal Adult Dog	F	6.8	57.3	Johnston (1985)
Normal Adult Dog	F	8.2	56.5	Johnston (1985)
Normal Adult Dog	F	9.1	84.8	Johnston (1985)
Normal Adult Dog	F	10.5	96.0	Johnston (1985)
Normal Adult Dog	F	12.3	94.0	Johnston (1985)
Mongrel Dog (N=10)	F	12.5	55.0	Takeda (1995)
Normal Adult Dog	F	12.7	57.8	Johnston (1985)
Normal Adult Dog	F	15.9	86.5	Johnston (1985)
Normal Adult Dog	F	19.1	77.5	Johnston (1985)
Woman	F	74.0	40	Prasad (2005), Ogden (2004)
Man (N=109)	M	92.0	223	Kohler (2008)
African Lion (N=7)	M	200.0	300	Lueders (2012), Nowak (1999)
African Elephant (5 years)	F	803.8	680	Balke (1988), Krumrey (1968)
African Elephant (5 years)	F	803.8	800	Balke (1988), Krumrey (1968)
African Elephant (6 years)	F	933.1	840	Balke (1988), Krumrey (1968)
African Elephant (6 years)	F	933.1	760	Balke (1988), Krumrey (1968)
African Elephant (6 years)	F	933.1	680	Balke (1988), Krumrey (1968)
African Elephant (7 years)	F	1058.6	820	Balke (1988), Krumrey (1968)
African Elephant (9 years)	F	1300.3	980	Balke (1988), Krumrey (1968)
African Elephant (9 years)	F	1300.3	800	Balke (1988), Krumrey (1968)
African Elephant (9 years)	F	1300.3	940	Balke (1988), Krumrey (1968)
African Elephant (10 years)	F	1417.4	880	Balke (1988), Krumrey (1968)
African Elephant (10 years)	F	1417.4	1180	Balke (1988), Krumrey (1968)
African Elephant (10 years)	F	1417.4	880	Balke (1988), Krumrey (1968)
African Elephant (11 years)	F	1532.4	860	Balke (1988), Krumrey (1968)
African Elephant (14 years)	F	1866.7	830	Balke (1988), Krumrey (1968)
Elephant	F	2700.0	1200	Hildebrandt (2000), Nowak (1999)
African Elephant (23 years)	F	2802.4	900	Balke (1988), Krumrey (1968)
African Elephant (25 years)	F	3000.3	1100	Balke (1988), Krumrey (1968)
Elephant	M	5400.0	1000	Fowler (2006), Nowak (1999)

Table S3. Urethral diameter.

Animal	Sex	Mass (kg)	Diameter (mm)	Source
Wister Rat	F	0.2	0.8	Chang (2000)
Wister Rat (N=10)	F	0.3	0.6	de Souza (2008)
Hybrid Rat (12 months) (N=5)	F	0.3	1.4	Russell (1996), Tasaki (2009)
Hybrid Rat (32 months) (N=5)	F	0.3	1.5	Russell (1996), Tasaki (2009)
Adult Rat (N=176)	M	0.6	1.8	Kunststyr (1982), Perrin (2003)
Short Hair Cat (7 weeks) (N=6)	F	0.9	2.0	Root (1996), Sturman (1985)
Short Hair Cat (7 weeks) (N=6)	M	0.9	3.3	Root (1996), Sturman (1985)
Short Hair Cat (7 months) (N=5)	F	2.4	3.2	Root (1996), Scott (1970)
Short Hair Cat (7 months) (N=5)	M	3.5	5.4	Root (1996), Lein (1983)
Woman	F	74.0	6.0	Gray (1918), Ogden (2004)
Man	M	86.1	6.0	Gray (1918), Ogden (2004)
Man (71.7 years) (N=32)	M	87.1	7.3	Tsujimoto (2003), Ogden (2004)
Miniature Horse (N=7)	M	102.5	15.0	Pozor (2002)
Pony (N=27)	M	171.5	19.0	Pozor (2002)
Hereford X Angus Bull (N=96)	M	300.0	6.5	Bailey (1975)
Light Horse (N=53)	M	452.5	26.0	Pozor (2002)
Heavy Horse (N=15)	M	687.0	30.0	Pozor (2002)
Elephant	F	2700.0	35.0	Hildebrandt (2000), Nowak (1999)
African Elephant (N=6)	M	4000.0	45.0	Hildebrandt (1998)
Asian Elephant (N=2)	M	5000.0	52.5	Hildebrandt (1998)
Asian Elephant	M	5000.0	48.0	Hildebrandt (1998)
Asian Elephant	M	5400.0	180.0	Fowler (2006)

Table S4. Shape factor, bladder capacity, and bladder pressure.

Animal	Sex	Mass (kg)	Shape factor α	Source
Mouse	F	0.03	0.25	Treuting (2011)
Rat	F	0.28	0.24	Praud (2003)
dog	M	11.8	0.17	Caceci, Johnston (1985)
White Pig (N=4)	F	70	0.22	Dass (2001)
Man	M	86.1	0.14	Skarva, Ogden (2004)

Animal	Sex	Mass (kg)	Bladder capacity (mL)	Source
Mouse (N=10)	F	0.02	0.15	Pandita (2000)
Mouse	M	0.03	0.22	Birder (2002)
Sprague Dawley Rat	M	0.25	1	Herrera (2010)
Cat (N=3)	F	3	5.4	Thor (1995)
Applehead Chihuahua	M	3.45	4.2	Experiment at local park
Mongrel Dog (N=14)	M	22.5	327	Abdel (2001)
Dog (N=52)	-	28.7	45.5	Atalan (1998)
Equine	-	500	4250	Higgins (2006)
Elephant	-	5000	18000	Fowler (2006), Shoshani (1982)

Animal	Sex	Mass (kg)	Bladder pressure (kPa)	Source
Mice (N=10)	F	0.02	6.25	Pandita (2000)
Sprague Dawley Rat (N=7)	F	0.24	6.03	Ishizuka (1996)
Wister Rat (N=3)	M	0.45	4.02	Van (1995)
Sprague Dawley Rat (N=18)	M	0.54	4.40	Schmidt (2003)
Cat (N=3)	F	3	4.31	Thor (1995)
Cat (N=4)	F	3.5	5.54	Walter (2005)
Dog	F	10	4.88	Hinman (1971)
Man (N=17)	M	75.5	5.78	Schmidt (2003)

Table S5. Urine flow rate.

Animal	Sex	Mass (kg)	Flow rate (mL/s)	Source
Rat 1	F	0.24	1.11	Experiment at Georgia Tech
Rat 5	F	0.25	2.39	Experiment at Georgia Tech
Rat 3	F	0.26	0.38	Experiment at Georgia Tech
Rat 4	F	0.27	1.43	Experiment at Georgia Tech
Rat 2	F	0.29	0.67	Experiment at Georgia Tech
Cat (N=4)	F	3.5	2.40	Walter (2005)
Woman (3-4 years)	F	16.60	8.42	Segura (1997), Ogden (2004)
Woman (5-6 years)	F	21.46	9.43	Segura (1997), Ogden (2004)
Woman (7-8 years)	F	28.81	10.26	Segura (1997), Ogden (2004)
Woman (9-11 years)	F	41.28	13.49	Segura (1997), Ogden (2004)
Woman (12-14 years)	F	56.57	14.79	Segura (1997), Ogden (2004)
Dog	F	18.14	17.07	Experiment at local park
Woman (59 years) (N = 183)	F	76.90	9.4	Madersbacher (1998) , Ogden (2004)
Woman (55 years) (N = 185)	F	76.90	20.10	Nitti (1999) , Ogden (2004)
Cow	F	635.00	450.00	L. Ely, University of Georgia
Cow	F	635.00	453.00	L. Ely, University of Georgia
Wister Rat (N=2)	M	0.45	0.15	Van (1995)
Sprague Dawley Rat (N=18)	M	0.54	0.13	Schmidt (2003)
Dunkin Hartley Guinea Pig (N=4)	M	0.94	0.27	Van (1995)
Applehead Chihuahua	M	3.45	1.04	Experiment at local park
Man (3-4 years)	M	17.1	7.90	Segura (1997), Ogden (2004)
Man (5-6 years)	M	22.5	8.43	Segura (1997), Ogden (2004)
Man (7-8 years)	M	30.1	9.30	Segura (1997), Ogden (2004)
Man (9-11 years)	M	39.5	10.10	Segura (1997), Ogden (2004)
Man (12-14 years)	M	55.8	12.76	Segura (1997), Ogden (2004)
Nigerian Dwarf Goat	M	37.8	3.11	Experiment at Zoo Atlanta
Nubian Goat	M	71.00	6.25	Experiment at Zoo Atlanta
Japanese Man (N=271)	M	63.30	16.70	Masumori (1996)
Man (N=17)	M	75.70	24.40	Schmidt (2003), Schmidt (2002)
American Man (N=467)	M	87.60	19.80	Masumori (1996)
Man (53.8 years) (N=58)	M	88.80	17.60	Folkestad (2004), Ogden (2004)

Table S6. Bibliography for images and animal masses.

Online Image	Source
Inset in Figure 1(c)	Man vyi. (2005) <i>Jersey cattle in Jersey</i> , Wikimedia Commons http://commons.wikimedia.org/wiki/File:Jersey_cattle_in_Jersey.jpg
Inset in Figure 2(b)	Stephens, J. (1992) <i>Lobund-Wistar Rat</i> https://visualsonline.cancer.gov/details.cfm?imageid=2568
Animal silhouettes in Figure 1(h)	Public Domain Pictures http://www.publicdomainpictures.net/
Histology in Table S4	Caceci, T. (2008) <i>Canine penis; H&E stain, paraffin section (decalcified), 20x</i> , VM8054: Veterinary Histology http://www.vetmed.vt.edu/education/curriculum/vm8304/lab_companion/histo-path/vm8054/labs/Lab23/EXAMPLES/EXURETH.HTM
Histology in Table S4	Skarva, F. <i>Cross-Section of a Normal Human Penis Showing the Urethra and Corpora Spongiosum, H&E Stain, LM X12</i> http://www.allposters.com/-sp/Cross-Section-of-a-Normal-Human-Penis-Showing-the-Urethra-and-Corpora-Spongiosum-H-E-Stain-LM-X12-Posters_i9005304_.htm

Body masses of animals considered

Nowak RM, Paradiso JL (1999) *Walker's mammals of the world* (The Johns Hopkins University Press, Baltimore, MD) Vol. 1, 6th Ed.

Mattern MY, McLennan DA (2000) Phylogeny and speciation of felids. *Cladistics* 16(2):232–253.

Wilcox C (1997) *The Great Dane* (Capstone, North Mankato, MN).

Wilson D, Burnie D (2001) *Animal: the definitive visual guide to the world's wildlife* (DK Publishing, New York).

Miller-Schroeder P (1997) *Gorillas* (Raintree Steck-Vaughn, Austin, TX).

Brown G (1996) *The Great Bear Almanac* (Globe Pequot, Guilford, CT).

Starkey P, Mwenya E, Stares J (1994) *Improving animal traction technology*. Proceedings of the First Workshop of the Animal Traction Network for Eastern and Southern Africa held 18-23 January 1992, Lusaka, Zambia (Technical Centre for Agricultural and Rural Cooperation, Wageningen, The Netherlands).

Lynette R (2013) *South American Tapirs* (Bearport Publishing, New York).

Linnaeus C (1758) *Systema naturae per regna tria naturae, secundum classes, ordines, genera, species, cum characteribus, differentiis, synonymis, locis*. (Laurentii Salvii, Stockholm), 10th Ed.

Kingdon J (1988) *East African Mammals: An Atlas of Evolution in Africa, Part A: Carnivores* (University of Chicago Press, Chicago) Vol. 3.

Hall MH, Comerford PM (1992) Pasture and hay for horses. *Cooperative Extension, The Pennsylvania State University* 32:1–4.

Bongianni M (1988) *Simon and Schuster's guide to horses and ponies of the world* (Simon and Schuster, New York).

Potts S (1997) *The American Bison* (Capstone, Mankato, MN).

Toon A, Toon S (2002) *Rhinos* (Voyageur Press, Minneapolis, MN).

Shoshani J, Eisenberg JF (1982) *Elephas maximus*. *Mammalian Species* 182:1–8.

Ogden CL, Fryar CD, Carroll MD, Flegal KM (2004) *Mean Body Weight, Height, and Body Mass Index: United States 1960-2002* (Department of Health and Human Services, Centers for Disease Control and Prevention, National Center for Health Statistics).

Krumrey WA, Buss IO (1968) Age estimation, growth, and relationships between body dimensions of the female African elephant. *Journal of Mammalogy* 49(1):22–31.

Tasaki M, et al. (2009) Simultaneous induction of non-neoplastic and neoplastic lesions with highly proliferative hepatocytes following dietary exposure of rats to tocotrienol for 2 years. *Archives of Toxicology* 83(11):1021–1030.

Perrin D, Soulage C, Pequignot J, Geloën A (2003) Resistance to obesity in Lou/C rats prevents ageing-associated metabolic alterations. *Diabetologia* 46(11):1489–1496.

Sturman J, Moretz R, French J, Wisniewski H (1985) Postnatal taurine deficiency in the kitten results in a persistence of the cerebellar external granule cell layer: correction by taurine feeding. *Journal of Neuroscience Research* 13(4):521–528.

Scott P, Hafez E (1970) *Reproduction and Breeding Techniques for Laboratory Animals*. (Lea and Febiger, Philadelphia).

Lein D, Concannon P (1983) Infertility and fertility treatments and management in the queen and tomcat. *Current Therapy VIII*. Kirk, R. (ed). (Elsevier Saunders, Philadelphia) 936–987.

Schmidt F, Shin P, Jorgensen TM, Djurhuus JC, Constantinou CE (2002) Urodynamic patterns of normal male micturition: Influence of water consumption on urine production and detrusor function. *The Journal of Urology* 168(4 Pt 1):1458–1463.